Eur. Phys. J. D **12**, 351–354 (2000)

THE EUROPEAN PHYSICAL JOURNAL D

EDP Sciences

Electron captures by positrons from hydrogenic ions in nonideal classical plasmas

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Received 21 January 2000 and Received in final form 27 April 2000

Abstract. In nonideal classical plasmas, the electron captures by positrons from hydrogenic ions are investigated. An effective pseudopotential model taking into account the plasma screening effects and collective effects is applied to describe the interaction potential in nonideal plasmas. The classical Bohr-Lindhard model has been applied to obtain the electron capture radius and electron capture probability. The modified hyperbolic trajectory method is applied to the motion of the projectile positron in order to visualize the electron capture probability as a function of the impact parameter, nonideal plasma parameter, projectile velocity, and plasma parameters. The results show that the electron capture probability in nonideal plasmas is always greater than that in ideal plasmas descried by the Debye-Hückel potential, *i.e.*, the collective effect increases the electron capture probability. It is also found that the collective effect is decreased with increasing the projectile velocity.

PACS. 52.20.-j Elementary processes in plasma

1 Introduction

Electron capture process in atom-charged particle collisions has been of great interest since this process is one of the basic processes in atomic collision physics [1]. The electron capture processes have been investigated widely using various methods [2–6] depend on the state of the projectile and target system. Collision processes involving the positrons and electrons have received many attentions since theses processes have many applications in atomic, plasma physics, and astrophysics. Especially, the annihilation of positrons has been widely investigated in astrophysical plasmas [7–9]. The theoretical investigations for positron-electron direct annihilation and positron formation also have been widely performed. Recently, in weakly coupled ideal plasmas, the electron capture by positron from target atom was investigated using the Bohr-Lindhard (BL) model with the Debye-Hückel potential [10]. When the relative interaction velocity v_{P} of the projectile is greater than the ground state orbital velocity v_Z $(= Ze^2/\hbar)$ of the hydrogenic ion with nuclear charge Z, *i.e.*, intermediate and high energy projectiles, the classical BL model has been known to be quite reliable since the de Broglie wave length of the projectile is smaller than the collision diameter for the capture interaction [3,4]. The Debye-Hückel effective potential describes the properties of a low density plasma and corresponds to a pair correlation approximation. The plasmas descried by the Debye-Hückel model can be called the ideal plasmas since the average energy of interaction between particles is small compared to the average kinetic energy of a particle [11]. However, multiparticle correlation effects caused by simultaneous interaction of many particles should be taken into account with increasing the plasma density. It is necessary to take into account not only short-range collective effects but also long range effects in the case of a plasma with a moderate density and temperature. In this case, the interaction potential cannot be described by the Debye-Hückel model because of the strong collective effects of nonideal particle interaction [12]. Then, the electron capture probability by the positron projectile from the hydrogenic ion target in nonideal plasmas would be different from that in ideal plasmas. Thus, in this paper we investigate electron capture processes in positron-hydrogenic ion collisions in nonideal plasmas. A pseudopotential model including the plasma screening effects and collective effects is applied to describe the interaction potential between the projectile positron and the target ion in nonideal plasmas. The classical BL model has been applied to obtain the electron capture radius and electron capture probability. The modified hyperbolic trajectory method is applied to the motion of the projectile positron in order to visualize the electron capture probability as a function of the impact parameter, nonideal plasma parameter, projectile velocity, and plasma parameters.

In Section 2, we derive the electron capture radius and electron capture cross-section by positrons from hydrogenic ions in nonideal plasmas using the BL model and the modified hyperbolic orbit trajectory method with the pseudopotential model including the plasma screening

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effects and collective effects. In Section 3, we obtain the scaled electron capture probability as a function of the impact parameter, nonideal plasma parameter, projectile velocity, and plasma parameters. We also investigate the variation of the scaled electron capture probability with changing the scaled modified impact parameter. The results show that the electron capture probability in nonideal plasmas is found to be always greater than that in ideal plasmas, *i.e.*, the collective effect increases the electron capture probability. It is also found that the collective effect is decreased with increasing the velocity of the projectile positron. Finally, in Section 4, a discussion is given.

2 Electron capture cross-section

Using the classical trajectory method, the electron capture cross-section can be given by [5]

$$\sigma_{\mathrm{C}} = 2\pi \int \mathrm{d}b\, b\, P_{\mathrm{c}}(b), \tag{1}$$

where b is the impact parameter and $P_{\mathrm{c}}(b)$ is the electron capture probability. In the BL model, the electron capture process happens when the distance between the projectile and a released electron is smaller than the electron capture radius R_{c} [2–4]. This electron capture radius is determined by equating the kinetic energy of the released electron in the frame of the projectile and the binding energy provided by the projectile. In the classical BL model, the electron capture probability is defined by the ratio of the collision time to the electron orbital time

$$P_{\mathrm{c}}(b) = \int_{-t_{\mathrm{c}}}^{t_{\mathrm{c}}} \frac{\mathrm{d}t}{\tau}, \tag{2}$$

where $t = 0$ is arbitrary chosen as the instant at which the projectile positron makes its closest approach to the target ion and t_{c} is the electron capture time within the electron capture radius. Here, the electron orbital time τ in the ground state of the hydrogenic ion with nuclear charge Z is given by

$$\tau = a_Z / \upsilon_Z, \tag{3}$$

where a_Z $(= a_0/Z = \hbar^2/me^2Z^2)$ is the first Bohr radius. For intermediate and high projectile velocities $\upsilon_{\mathrm{P}} \geq \upsilon_Z$, this classical expression of the electron capture cross-section (Eq. (1)) is known to be valid [3,4].

In a recent paper [12], an integro-differential equation for the effective potential of the particle interaction taking into account the simultaneous correlations of N particles was obtained on the basis of a sequential solution of the chain of Bogolyubov equations for the equilibrium distribution function of particles of a classical nonideal plasma and an analytic expression for the pseudopotential of the particle interaction in a nonideal plasmas was also obtained by application of the spline-approximation. Using the pseudopotential taking into account the plasma screening effects and collective effects, the interaction potential $V(\mathbf{r})$ between the projectile positron and the target ion with charge Z in nonideal plasmas can be represented by

$$V(\mathbf{r}) = \frac{Ze^2}{r} \mathrm{e}^{-r/\Lambda} \frac{1 + \gamma f(r)/2}{1 + c(\gamma)}, \tag{4}$$

where $\mathbf{r}$ is the position vector of the projectile positron from the target ion, Λ is the Debye length,

$$f(r) = (\mathrm{e}^{-\sqrt{\gamma} r/\Lambda} - 1)(1 - \mathrm{e}^{-2r/\Lambda})/5$$

and γ $(\equiv e^2/\Lambda k T_{\mathrm{e}})$ is the nonideal plasma parameter,

$$c(\gamma) \cong -0.008617 + 0.455861\gamma - 0.108389\gamma^2 + 0.009377\gamma^3$$

is the correlation coefficient for different values of γ, and T_{e} is the electron temperature. When $\gamma \ll 1$, *i.e.*, weakly nonideal or rare ideal plasmas, the pseudopotential (Eq. (4)) goes over into the Debye-Hückel potential $V(\mathbf{r}) \rightarrow (Ze^2/r)\mathrm{e}^{-r/\Lambda}$. Using this pseudopotential, the electron capture radius R_{c} in nonideal plasmas can be obtained by

$$\frac{e^2}{R_{\mathrm{c}}} \mathrm{e}^{-R_c/\Lambda} \frac{1 + \gamma f(R_{\mathrm{c}})/2}{1 + c(\gamma)} \cong \frac{1}{2}\mu \upsilon_{\mathrm{P}}^2, \tag{5}$$

since the kinetic energy of the released electron in the frame of the projectile positron has to be smaller than the binding energy provided by the projectile positron, where μ $(= m/2)$ is the reduced mass and m is the electron mass. After some algebra using the perturbation calculation since the electron capture radius R_{c} is usually smaller than the Debye length Λ, the electron capture radius including the plasmas screening effect and collective effect is found to be

$$\frac{R_{\mathrm{c}}}{\Lambda} \cong \frac{R_0/\Lambda}{1 + R_0/\Lambda + c(\gamma)} + \left(\frac{R_0}{\Lambda}\right)^3 \frac{(1/2 - \gamma^{3/2}/5)}{[1 + R_0/\Lambda + c(\gamma)]^3}, \tag{6}$$

where $R_0 \equiv 4e^2/m\upsilon_{\mathrm{P}}^2$.

For projectiles such as electrons and positrons rather than heavy nuclei, the curved trajectory method must be applied to describe the projectile motion in Coulomb fields because of the small mass ratios $(m/M \ll 1)$. For positron projectiles, the useful parametric representation [13] of the hyperbolic orbit trajectory for $\mathbf{r}(t)$ in x-y plane is given by

$$\begin{aligned} x &= d\,(\varepsilon^2 - 1)^{1/2} \sinh w, \\ y &= d\,(\cosh w - \varepsilon), \\ r(t) \equiv |\mathbf{r}(t)| &= d(\varepsilon \cosh w + 1), \\ t &= \frac{d}{\upsilon_{\mathrm{P}}}(\varepsilon \sinh w + w), \quad -\infty < w < \infty \end{aligned} \tag{7}$$

where d and ε are half of the distance of closest approach in a head-on collision and the eccentricity, respectively. Including the plasma screening effects and collective effects, the parameters d and ε are given by

$$\frac{d}{\Lambda} \cong \frac{d_0/\Lambda}{1+d_0/\Lambda+c(\gamma)} + \left(\frac{d_0}{\Lambda}\right)^3 \frac{(1/2-\gamma^{3/2}/5)}{[1+d_0/\Lambda+c(\gamma)]^3}, \quad (8)$$

$$\varepsilon = (1+b^2/d^2)^{1/2}, \quad (9)$$

where $d_0 \equiv Z_{\mathrm{eff}} e^2/m v_{\mathrm{P}}^2$ and Z_{eff} is the effective charge of the target ion seen by the projectile positron, for example, $Z_{\mathrm{eff}} = Z - 5/16$ for one-electron atoms. The hyperbolic orbit trajectory method has been widely used to investigate the electron-ion collisional impact excitation [14,15] and bremsstrahlung [16] processes. Using equations (2, 7), the electron capture probability using the hyperbolic orbit trajectory method is given by

$$P_{\mathrm{c}}(b) = \frac{2d}{\tau v_{\mathrm{P}}}(\varepsilon \sinh w_{\mathrm{c}} + w_{\mathrm{c}}), \quad (10)$$

where the parameter w_{c} is determined by the electron capture time t_{c}:

$$w_{\mathrm{c}} = \ln\left\{\frac{(R_{\mathrm{c}}/d-1)}{(1+b^2/d^2)^{1/2}} + \left[\frac{(R_{\mathrm{c}}/d-1)^2}{(1+b^2/d^2)} - 1\right]^{1/2}\right\}. \quad (11)$$

From equation (11), the maximum impact parameter $b_{\max}$ is given by $(R_{\mathrm{c}}^2 - 2R_{\mathrm{c}}d)^{1/2}$. However, this is physically unreasonable since the electron capture process is also possible in the region $(R_{\mathrm{c}}^2 - 2R_{\mathrm{c}}d)^{1/2} \le b \le R_{\mathrm{c}}$. Thus, we have to modify the impact parameter in order to obtain the correct electron capture probability and electron capture cross-section. In the Bohr-Lindhard model, the electron release radius R_{r} is taken to be that the initial electron energy should be equal to the height of the electrostatic potential barrier given by the total potential energy acting on the electron. For collisions between the projectile ion with charge z and hydrogenic ion target with charge Z, the electron release radius R_{r} is given by $2(1+2\sqrt{z/Z})a_Z$, which is the independent of the projectile velocity. When $R_{\mathrm{c}} > R_{\mathrm{r}}$, *i.e.*, low velocities, the electron capture cross-section is geometrical with the maximum of the cross-section $\sigma_{\mathrm{C}} = \pi R_{\mathrm{r}}^2$ [17]. For high velocities, the electron capture process only happens when the distance between the projectile and a release electron is smaller than the electron capture radius R_{c} [4]. Physically, the maximum impact parameter should be equal to the electron capture radius R_{c} since the electron capture probability has been defined by the electron capture time. Since the electron capture process is only proceeded within the electron capture sphere with the radius R_{c}, after some straightforward manipulations, the modified impact parameter b' can be obtained by the shortest distance from the tangent [16] at the position $[x(w_{\mathrm{c}}),\ y(w_{\mathrm{c}})]$:

$$b' = d(\varepsilon^2-1)^{1/2}\left(\frac{\varepsilon\cosh w_{\mathrm{c}}+1}{\varepsilon\cosh w_{\mathrm{c}}-1}\right)^{1/2}, \quad (12a)$$

$$= b(1-2d/R_{\mathrm{c}})^{-1/2}. \quad (12b)$$

From equations (11, 12b), we can readily show that the modified maximum impact parameter becomes $b'_{\max} = R_{\mathrm{c}}$. For $b' > R_{\mathrm{c}}$, the trajectory of the projectile would be a straight line since the capture probability is zero for $t > |t_{\mathrm{c}}|$. However, for $b' \le R_{\mathrm{c}}$, *i.e.*, $t \le |t_{\mathrm{c}}|$, the trajectory of the projectile would be then the screened hyperbolic orbit. Hence, our modification on the impact parameter is quite reliable and can be applied to investigate the electron capture probability.

3 Electron capture probability

From equations (10, 11, 12b), the electron capture probability as a function of the modified impact parameter is given by

$$P_{\mathrm{c}}(b') = \frac{2d}{\tau v_{\mathrm{P}}}\left\{\left[\frac{R_{\mathrm{c}}}{d}\left(\frac{R_{\mathrm{c}}}{d}-2\right)\left(1-\frac{b'^2}{R_{\mathrm{c}}^2}\right)\right]^{1/2} - \frac{1}{2}\ln\left[1+\frac{R_{\mathrm{c}}}{d}\left(\frac{R_{\mathrm{c}}}{d}-2\right)\frac{b'^2}{R_{\mathrm{c}}^2}\right] + \ln\left[\left(\frac{R_{\mathrm{c}}}{d}-1\right)+\left[\frac{R_{\mathrm{c}}}{d}\left(\frac{R_{\mathrm{c}}}{d}-2\right)\left(1-\frac{b'^2}{R_{\mathrm{c}}^2}\right)\right]^{1/2}\right]\right\}. \quad (13)$$

Then, the electron capture cross-section (Eq. (1)) can be rewritten as

$$\sigma_{\mathrm{C}} = 2\pi R_{\mathrm{c}}^2 \int \mathrm{d}\tilde{b}'\, \tilde{b}'\, P_{\mathrm{c}}(\tilde{b}'), \quad (14)$$

where $\tilde{b}'$ $(\equiv b'/R_{\mathrm{c}})$ the scaled modified impact parameter. Thus, the scaled differential electron capture cross-section in units of πR_{c}^2 becomes

$$\frac{\mathrm{d}\sigma_{\mathrm{C}}/\mathrm{d}\tilde{b}'}{\pi R_{\mathrm{c}}^2} = 2\tilde{b}' P_{\mathrm{c}}(\tilde{b}') \quad (15a)$$

$$= 2\left(\frac{2R_{\mathrm{c}}}{\tau v_{\mathrm{P}}}\right)\tilde{b}'\tilde{P}_{\mathrm{c}}(\tilde{b}', R_{\mathrm{c}}/d). \quad (15b)$$

Here, $\tilde{b}'\tilde{P}_{\mathrm{c}}(\tilde{b}', R_{\mathrm{c}}/d)$ is the scaled electron capture probability including the plasma screening effects and collective effects through the radio R_{c}/d (Eqs. (6, 8)).

In order to investigate the plasma screening effects and collective effects on the electron capture probability by the positron projectile we consider the two cases of the projectile velocity $v_{\mathrm{P}}/v_Z = 1$ and 3 since the classical BL model is known to be valid for intermediate and high energy projectiles [3,4] and we choose $a_\Lambda = 0.1$ and $\gamma = 1$. Figures 1 and 2 show the scaled electron capture probabilities $\tilde{b}'\tilde{P}_{\mathrm{c}}(\tilde{b}', R_{\mathrm{c}}/d)$ by the positron projectile from the hydrogenic ion target with nuclear charge $Z = 2$ as functions of the scaled impact parameter including the plasma screening effects and collective effects for $v_{\mathrm{P}}/v_Z = 1$ and 3, respectively. As we can see in these figures, the electron capture probability in nonideal plasmas ($\gamma = 1$) described by the pseudopotential is always greater than that in ideal plasmas ($\gamma = 0$) descried by the Debye-Hückel potential.

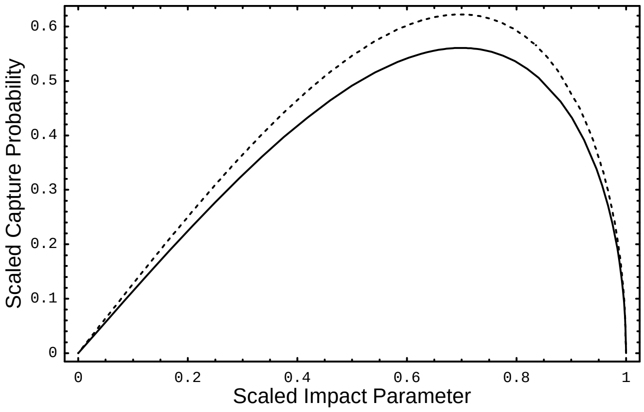

Fig. 1. The scaled electron capture probability $\tilde{b}'\tilde{P}_c(\tilde{b}', R_c/d)$ by the positron from the hydrogenic ion with nuclear charge $Z = 2$ as a function of the scaled impact parameter $\tilde{b}'$ ($\equiv b'/R_c$) for $a_\Lambda = 0.1$ and $v_P/v_Z = 1$. The solid line represents the electron capture probability in ideal plasmas, *i.e.*, $\gamma = 0$. The dotted line represents the electron capture probability in nonideal plasmas with $\gamma = 1$.

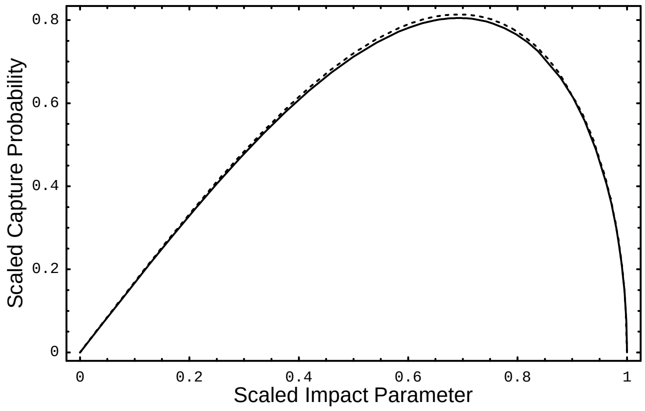

Fig. 2. The scaled electron capture probability $\tilde{b}'\tilde{P}_c(\tilde{b}', R_c/d)$ by the positron from the hydrogenic ion with nuclear charge $Z = 2$ as a function of the scaled impact parameter $\tilde{b}'$ ($\equiv b'/R_c$) for $a_\Lambda = 0.1$ and $v_P/v_Z = 3$. The solid line represents the electron capture probability in ideal plasmas, *i.e.*, $\gamma = 0$. The dotted line represents the electron capture probability in nonideal plasmas with $\gamma = 1$.

Thus, it is found that the nonideality ($\gamma \neq 0$) of plasmas, *i.e.*, the collective effect, increases the electron capture probability. It is also found that the collective effect is decreased with increasing the projectile velocity.

4 Discussion

We investigate the plasma screening effects and collective effects on the electron captures by positrons from hydrogenic ions in nonideal classical plasmas. An effective pseudopotential model taking into account the plasma screening effects and collective effects is applied to describe the interaction potential in nonideal plasmas. The classical Bohr-Lindhard model has been applied to obtain the electron capture radius and electron capture probability using the modified hyperbolic trajectory method. The scaled electron capture probability by positrons from hydrogenic ions is obtained as a function of the impact parameter, nonideal plasma parameter, projectile velocity, and plasma parameters. It is found that the electron capture probability in nonideal plasmas is always greater than that in ideal plasmas descried by the Debye-Hückel potential, *i.e.*, the collective effect increases the capture probability. It is important to note that the collective effect is decreased with increasing the velocity of the projectile positron. These results provide a useful information of the electron capture processes by positrons from target ions in nonideal classical plasmas.

The author gratefully acknowledges Prof. Richard M. More and Dr. Izumi Murakami for useful comments and stimulating discussions. The author would like to thank the anonymous referee for suggesting improvements to this text. This work was supported by the Korean Ministry of Education through the Brain Korea (BK21) Project, by the Korea Basic Science Institute through the HANBIT User Development Program (FY2000), and by Hanyang University, South Korea, made in the program year of 2000.

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